PARETO EFFICIENCY ANALYSIS OF PRIVATE TOLL ROAD MARKETS:

Leveraging PPPs to finance investments in infrastructure

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Moreover, data (American Society of Civil Engineers, 2013) shows 32% of America’s major roads are in poor or mediocre condition.
Key issues

• Most of public road projects in the US are funded by Highway trust fund.
• According to the Congressional Budget Office (CBO), from 2021 to 2026 trust fund revenue is projected to total $243 billion, but outlays will amount to $364 billion, resulting in an imbalance of 121 Billion.
• Insolvency issue of Highway trust fund brings difficulty in investment to new roads.
Private participation in roads

• In many Europe countries, a significant (37%) percentage of road lengths are under concession, and most of them are operated by private firms. (Albalate et al. 2009)
• Private investments to roads in China, Guangzhou-Shenzhen super-highway project in south of China as an example of Built-operate transfer in China.
• In 2005, Chicago Skyway concession became the first privatization of existing road in the US.
Different objectives

• Government’s objective is to maximize social welfare, which is commonly assumed to be aggregate surplus.

\[ SW = \int_0^N P(n)dn - N \cdot C(N, K) - C^c(K) \]

• Private firms’ objective is to maximize profit:

\[ \pi = N \cdot \tau - C^c(K) \]

where \( P(n) \) is the inverse demand function, \( N \) is the number of users or volume, \( K \) is the capacity of road and \( C(N,K) \) is the travel cost function, which has been converted to monetary cost in order to compare with toll. \( \tau \) is the toll, and \( C^c(K) \) is the construction cost function.
Demand in a single road

Inverse demand determines willingness to pay for the trip. In equilibrium, willingness to pay equals to total cost, the sum of travel time cost and toll.

\[ P(N) = \tau + C(N, K) \]

Taking derivative of both side with respect to \( \tau \), and rewrite the expression:

\[ \frac{\partial N}{\partial \tau} = \frac{1}{P' - C_N} \]

Similarly for capacity \( K \):

\[ \frac{\partial N}{\partial K} = \frac{C_K}{P' - C_N} \]

where \( C_N \) is the partial derivative of average congestion with respect to number of users, namely, average congestion externalities and \( P' \) is the derivative of inverse demand function, \( C_K \) is the partial derivative of average congestion with respect to capacity \( K \).
Assumptions

- First-order (strict preference):
  \( P' < 0, \ C_N > 0, \ C_K < 0 \) therefore \( P' - C_N < 0 \).

\[
\frac{\partial N}{\partial K} > 0 \ , \ \frac{\partial N}{\partial \tau} < 0.
\]

\[
\frac{\partial N}{\partial K} \ C_N + C_K > 0.
\]
Assumptions

• Second-order (concavity):
Assume that construction cost is constant return to scale.
\( N(K, \cdot) \) is concave. i.e. the marginal benefit is decreasing.
\( N(\cdot, \tau) \) is concave. i.e. the marginal disutility is increasing.
Congestion technology is convex and homogeneous of degree 0, that is
\( C(tN, tK) = C(N, K) \). In transportation literature, we call \( \frac{N}{K} \) the volume-capacity ratio.
Therefore \( C(N, K) = g \left( \frac{N}{K} \right) \), where g is convex.

• It can be shown that given the above assumptions with a “rational” toll, the toll which leads to nonzero demand, social welfare and profit function are concave.
• Given the concavity, we are able to use first order condition to seek for maximizer.
Marginal pricing in single road – government

- Taking derivative of social welfare with respect to toll,

\[
\frac{\partial SW}{\partial \tau} = \frac{\partial N}{\partial \tau} [\tau - N \cdot C_N] = \frac{\tau - N \cdot C_N}{P' - C_N}
\]

where \( \tau - N \cdot C_N \) is internalized congestion externalities. FOC implies that \( \tau - N \cdot C_N = 0 \), that is government uses toll to internalize congestion externalities. The above toll is referred to as “Pigovian toll”.

- Same for capacity,

\[
\frac{\partial SW}{\partial K} = \frac{C_K}{P' - C_N} [\tau - N \cdot C_N] - N \cdot C_K - MC^c(K)
\]

If we plug in \( \tau = N \cdot C_N \), that is Pigovian toll is charged, FOC implies that \(-N \cdot C_K - MC^c(K) = 0\). This result is consistent with the first best pricing in Verhoef’s paper (2007), in which Lagrangian method is used for solving FOC.
Marginal pricing in single road – firms

- Taking derivative of profit with respect to toll and capacity

\[
\frac{\partial \pi}{\partial \tau} = \frac{\partial N}{\partial \tau} (\tau - N \cdot C_N + N \cdot P')
\]

\[
\frac{\partial \pi}{\partial K} = \frac{\partial N}{\partial K} \tau - MC^e[K]
\]

The optimal toll for private sector is \( N \cdot C_N - N \cdot P' \). That is, in addition to charge Pigovian toll, private sector has incentive to charge a markup \(-N \cdot P'\), which depends on demand elasticity. Private firm only cares on the effect of capacity on demand and therefore effect on their profit.
Marginal pricing in single road – firms

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Would competition or ownership regime change that?
Roads under different ownership regime

- De Palma and Lindsey (2000) compares welfare gain of private tolling under 4 different ownership regime, public-free, private-free, private-private and public-private in a 2-route parallel network.
- Verhoef (2007) examine the second best road pricing when a private toll road competes against one unpriced public road as a substitute or a complement.
- Mun and Ahn (2008) also presents a similar study on a serial 2-road network.
- Inefficiency on social welfare is a big issue in private tolling.
Demand system of road network

• Demand system determines how total demand (for each OD pair) is distributed to each roads in a network. For any arbitrary road network, there is a corresponding system of equations that specifies the relationships between volume in each road.
• Such network structure can be used in other field, e.g. computer network, airline route design and power supply.
• In order to simplify our analysis, we focus on simple network structure, parallel and serial network.
Demand system of road network

• For parallel (or substitute) network,

\[ P\left( \sum_j N_j \right) = C(N_i, K_i) + \tau_i \forall i \]

There are \( N_i \) users in \( i \)th road with capacity \( K_i \) and toll \( \tau_i \). According to Wardrop's (1952) first Principle of equilibrium, all used roads should have equalized generalized cost, that is, sum of travel time cost and toll.

• For serial network,

\[ P(N) = \sum_i \left( C(N_i, K_i) + \tau_i \right) \forall i \]

For each OD pair, there is a common volume \( N \) in each road. And the generalized cost for each user is the sum of cost in each road.
Jacobi of demand system

• Take derivative of parallel demand system, we have:

\[ J^\tau = (1 \cdot P' - CN)^{-1} \]

\[ J^K = CK \cdot (1 \cdot P' - CN)^{-1} \]

where \( J^\tau \) and \( J^K \) are Jacobi matrix of N on toll and capacity, respectively. That is \( J^\tau_{ij} = \frac{\partial N_i}{\partial \tau_j} \) and \( J^K_{ij} = \frac{\partial N_i}{\partial K_j} \).

CN and CK is a diagonal matrix with \( \{ C_{N1}, C_{N2}, \ldots \} \) and \( \{ C_{K1}, C_{K2}, \ldots \} \) as diagonal elements.

• We can use the same trick for serial network and get much simpler result in which there is only one common volume N.
Marginal effect – road network

• For social welfare:

\[ \frac{\partial SW}{\partial \tau_i} = \sum_j J_{ij}^r [\tau_j - N_j \cdot CN_j] \]

\[ \frac{\partial SW}{\partial K_i} = \sum_j J_{ij}^K [\tau_j - N_j \cdot CN_j] - MC^e(K_i) - N_i \cdot CK_i \]

Similar to our result in single road case. But now there is a weighted internalized externalities term.

• For profit:

\[ \frac{\partial \pi_i}{\partial \tau_{ij}} = \begin{cases} J_{ij}^r \cdot \tau_i & \text{if } i \neq j \\ J_{ij}^r \cdot \tau_i + N_i & \text{if } i = j \end{cases} \]

\[ \frac{\partial \pi_i}{\partial K_j} = \begin{cases} J_{ij}^K \cdot \tau_i & \text{if } i \neq j \\ J_{ij}^K \cdot \tau_i + MC^e(K_i) & \text{if } i = j \end{cases} \]

Similarly, marginal effects on profits are essentially marginal effects on demand.
Pareto Optimum

• In most cases, social optimum is desired, i.e. the maximizer of aggregate surplus. However, there are concerns.
• Private firms lack incentive to invest on public infrastructure, due to risks and high capital cost. They have difficulty seeking enough profits at social optimum.
• In PPP, negotiations between government and private firms result in contracts that no better solution makes both parties better off.
• Due to exogenous uncertainties, social optimum decisions may vary. Then, in order to persuade private sectors to change, government have to think of a way to increase social welfare without hurting their profits.
• Those concerns leads to multi-objective programming, in which solution is referred to as Pareto Optimum.
Pareto Optimum – single road

• Generally Pareto optimal solution guarantees no other solution making both parties better off.

• Definition:
For single road, we say decision combination \( \{\tau^*, K^*\} \) is Pareto-optimal if and only if there is no other combination \( \{\tau, K \} \) such that \( SW(\tau, K) > SW(\tau^*, K^*) \) and \( \pi(\tau, K) > \pi(\tau^*, K^*) \). We say the set of such decision combinations Pareto frontier.
Numeric example

Settings:
BPR function
Linear demand
Linear construction cost
Pareto Optimum – multiple roads

• There are some variations for cases in which multiple parties are involved.
• Feldman (1973) defines Pareto optimality, pairwise optimality and core and their relationship in bilateral trading problem.
• Goldman (1982) follows his work and formalized the t-wise optimality as no weakly better deviation for any t traders, as a coalition.
• While we need a different one, because collusion between firms is strictly unfavorable.
Aggregate optimum

• If we believe that government makes trade off between social surplus and aggregate profit. As higher profitability leads to more private participants. Then, we can imagine there is a union of private firms as a party. We define the trade off between government and the union as aggregate optimum.

• The Pareto optimum between these two parties are no different from the single road case, except aggregate profit is used instead of single road profit.
Decentralized optimum

• If we assume that individual's benefits are taken into account, then, we can't improve aggregate profit by hurting any firm's benefit.
• Essentially, government negotiate with each private firm and therefore Pareto-optimality has to be achieved for each firm and government.
• We decentralize the bargaining power of union of private sectors into each individual firms.
• But again, we avoid collusion between any pair of two private firms.
Decentralized optimum

• Definition
There are N+1 self-interested agents in an economy, where one of them represents government and the rest N of them represent N private firms. We say decision combination \( \{ \tau^*, K^* \} \) is decentralized optimal if and only if, there is no decision combination for an individual firm \( i \) \( \{ \tau_i, K_i \} \) such that,

\[
SW (\tau_i, K_i, \tau^*_{-i}, K^*_{-i}) > SW (\tau^*_i, K^*_i, \tau^*_{-i}, K^*_{-i})
\]

\[
\pi_i (\tau_i, K_i, \tau^*_{-i}, K^*_{-i}) > \pi_i (\tau^*_i, K^*_i, \tau^*_{-i}, K^*_{-i}).
\]
Numeric example

Settings:
Two Symmetric roads
With capacity 500
BPR function
Linear demand
Linear construction cost
Two-stage Bertrand game

- Intuitively, negotiation results in Pareto Optimality, while strategic interactions between multiple agents may not lead to Pareto Optimum.
- In order to check Pareto Optimality of decisions that result from strategic behaviors of firms, we define a two-stage game in which each firm simultaneously choose capacity in first stage and then simultaneously choose toll in second stage.
- Such game is referred to as Bertrand game. De Borger (2006) generalized and applied this idea in private tolling market.
Tolling stage

- Following backward induction, we start by analyzing second stage. According to FOC, firm $i$ sets equilibrium toll $\tau_i^e$ as:

$$\tau_i^e = \frac{N_i}{-J_{ii}^T}$$

- Marginal effect of equilibrium toll $\tau_i^e$ on $i$ is 0 (envelop theorem), on $j \neq i$ is following:

$$\frac{\partial \pi_i(\tau^e)}{\partial \tau_j} = \tau_i^e \cdot J_{ij}^T = \frac{N_i \cdot J_{ij}^T}{-J_{ii}^T}$$

The sign of $J_{ii}^T$ is always negative, as strict preference assumption, but $J_{ij}^T$ depends on structure of network. In parallel network, $J_{ij}^T$ is always positive.
Tolling stage

- It can be shown that equilibrium tolls are trivially decentralized optimum. Because each firm maximize their profits given capacity and other’s strategies.
- But whether or not equilibrium tolls are aggregate optimum depends on network structure. To see this,

\[ \sum_i \frac{\partial \pi_i(\tau^e)}{\partial \tau_j} = \sum_{i \neq j} \frac{N_i \cdot J_{ij}^T}{-J_{ik}^T} \]
Capacity investment stage

• In capacity stage, each firm has already known the equilibrium tolling function $\tau_i^e$ derived in tolling stage. And we call $\frac{\partial \tau_i^e}{\partial K_i}$ the strategic effect. If firms build more capacity in first stage, they believe the additional capacity will change their equilibrium toll in second stage. If the additional capacity results in higher toll, then strategic effect is positive and vice versa.

• mathematically,

$$\text{sign}(\frac{\partial \tau_i^e}{\partial K_i}) = \text{sign}(N_i \cdot \frac{\partial J_i^e}{\partial K_i} - J_i^e \cdot J^K)$$
Capacity investment stage

• FOC for capacity is:

\[
\sum_j J_{ij} \cdot \frac{\partial \tau_j^c}{\partial K_i} + J_{ii}^K \cdot \tau_i^c + N_i \cdot \frac{\partial \tau_i^c}{\partial K_i} - MC^c(K) = 0
\]

The sign of \(J_{ii}^K\) is always positive, as strict preference assumption, but \(J_{ij}^K\) depends on structure of network.

• Recall marginal effect of capacity on profit, we can show

\[
\frac{\partial \pi_i(\tau_i^c)}{\partial K_i} = -N_i \cdot \frac{\partial \tau_i^c}{\partial K_i} - \left( \sum_j J_{ij} \cdot \frac{\partial \tau_j^c}{\partial K_i} \right) \cdot \tau_i^c
\]

The sign depends on strategic effect, so we can’t even guarantee the necessary conditions of decentralized optimum. If the sign is positive, we say firms are under-investing. And firms are over-investing if negative. However, in simultaneous tolling-capacity game, the marginal effect is always 0 as there is no strategic effect.
# Summary

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<thead>
<tr>
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<th>Decentralized optimum (necessary condition)</th>
<th>Aggregate optimum (necessary condition)</th>
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<tbody>
<tr>
<td></td>
<td>toll</td>
<td>capacity</td>
</tr>
<tr>
<td>parallel network</td>
<td>Yes</td>
<td>Depends on sign and magnitude of strategic effect</td>
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<tr>
<td>serial network</td>
<td>Yes</td>
<td>Depends on sign and magnitude of strategic effect</td>
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<tr>
<td>general network</td>
<td>Yes</td>
<td>Depends on sign and magnitude of strategic effect</td>
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Conclusion

• Generalization of demand system
We need to formalize a system of equations to specify demand-cost relationship between multiple roads in static User Equilibrium. These results can also be used in other network-related problem, i.e. airline route design, computer routing system.

• Marginal effect and Jacobi
Given the Jacobi derived from a demand system, our research shows the analysis on marginal effects on social welfare and profits of toll and capacity choice. The analysis doesn’t rely on a special network structure (parallel or serial).

• Aggregate optimum and decentralized optimum
We come up with two Pareto Optimal structure to characterize trade off between public benefits and private benefits, which can be used for design of subsidy mechanisms.
Thank you!

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